Math 2550 - Homework # 8

Linear Transformations and Eigenvalues

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1. Show that the following are linear transformations by finding a matrix A such that $T(\vec{v}) = A\vec{v}$.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ 3x - y \end{pmatrix}$

   (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ where $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - y + z \\ y - 4z \end{pmatrix}$

   (c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ x + y + z \\ 2x + z \end{pmatrix}$

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2. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

   Find a matrix A such that $T(\vec{v}) = A\vec{v}$.

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3. For each matrix A do the following: (i) Find the eigenvalues of A, (ii) Find a basis for each eigenspace $E_\lambda(A)$, (iii) For each eigenvalue, compute it's algebraic and geometric multiplicity.

   (a) $A = \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$

(c) $A = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

(d) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

(e) $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$

(f) $A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

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4. Let A be an $n \times n$ matrix. Suppose that λ is an eigenvalue of A with corresponding eigenvector $\vec{x}$. Find a formula for $A^n\vec{x}$ for $n = 1$ and $n = 2$ and $n = 3$. What is the formula for general n?
